Implication of Bayesian Technique to Improve Econometric Model Forecasts

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Abstract— This study presents the implication of Bayesian technique on simple statistical and econometric forecasting models to improve the forecast performances of the models. We consider a nonlinear, non-stationary time series of household electricity demand to demonstrate the Bayesian implication to statistical techniques. In this forecasting process, the electricity demand is considered a function of electricity price, household use of electric appliances, personal income, number of households, and urban conditions. We applied the Bayesian statistical technique on a multivariate linear regression model to predict the parametric values of the regression demand model. In the Bayesian process, the forecast is generated from the inferences of marginal posterior distribution of the model parameters obtained by Markov Chain Monte Carlo simulation process. Forecast result is tested and compared with actual data and two alternate models. The Bayesian model is proven to be an effective forecasting method with a great flexibility and capacity to solve multi-dimensional time series models by continuously updating the estimated parameter values as the demand changes over time. Test results indicate that Bayesian implication has significantly improved the performance of regression model. In the comparison, the error index reveal that Bayesian model outperform the forecast obtained by an artificial neural network model and a classical regression model. Application of Bayesian approach has proven to be efficient in predicting the trend changes and future proclivity of the large-scale multivariate time series data.

Index Terms— Bayesian technique, classical artificial neural network, multivariate linear regression, electricity load data, forecast validation, household electricity demand, Markov Chain Monte Carlo simulation, multivariate time series.

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1 INTRODUCTION

Econometric forecasting models follow several principles: keep the model simple, use as much data as one can get and use theory (not the data) as a guide to select casual variable [1]. Since the theories of econometric forecasting models are data driven, the values of the model parameters are static for a given set of data. Theories have little control to capture the dynamics behavior of the dataset to obtain the change of behavior of the input data. For example, statistical techniques such as multiple linear regression models use multiple input variables to predict random demand for the consumables, but the parameters of the models remain unchanged for a given set of data. A prediction model works well if the parameters of the model are dynamically updated as the demand changes and new information becomes available over time. The Bayesian technique is a statistical modeling approach recently attracted many researchers, which allows wide flexibility in the input variables and accommodates any changes of pattern or behavior of variables. The Bayesian method is increasingly applied to complex forecasting applications because of the advantages over existing statistical forecasting techniques due to the ability to react with any changes and the capacity to adjust with new occurrences outside of the routine model.

Accurate energy demand forecast is very important in developing countries for sustainable economic development. The study considers a real time series of household energy consumption data to demonstrate the benefit of Bayesian technique implied to econometric forecasting models. The selec-tion of residential electricity consumption data is a good choice to illustrate the capacity of Bayesian implication to capture nonlinear, non-stationary demand pattern. The recent data history indicates a dramatic increase of energy demand due to ongoing industrialization, economic growth and changes of demographic factors such as population and per capita income growth. If there is a change in the electricity consumption demand due to various reasons such as new urbanization, government policy or natural disasters; the subjective view can be incorporated in the model by using the judgment of experts' views about the new situation. Thereby, an immediate override can be made in Bayesian procedure to adjust the forecasts. When time series shows significant changes and alterations in their pattern, most models are not sufficient to pick up these changes.

In this paper, we have two goals. First, we present a Bayesian statistical technique implied to multivariate linear regression model to forecast long term residential electricity consumption demand of an industrially developing country. The second goal is to test the effectiveness of the Bayesian approach to regression model by comparing it with actual data and an alternate model. The forecast result of the proposed technique is compared with the forecast produced by classical regression model and artificial neural network model.

Hence the objective of our study is to demonstrate the application of Bayesian technique on an econometric forecasting model to improve the quality of the forecasts. The remainder

of this paper is organized as follows: in Section 2 the academic literature of energy forecasting models is presented. In Section 3, Bayesian forecasting method is overviewed for implication, where possible, the econometric forecasting models. In Section 4, we present the proposed forecasting model using a case study. In Section 5, the forecast results of the proposed Bayesian model are presented. In Section 6, the empirical performance of forecasting models is evaluated. The implication of Bayesian model and conclusion are presented in Section 7.

2 LITERATURE ON ENERGY DEMAND FORECASTING

Electricity usage is essential for continuous economic development and urbanization. Long term projection of residential electricity demand is vital for decision makers to develop strategic resource planning and energy policy. Developing countries cautiously focus on predicting future energy demand plan in order to propose optimal capital investment decisions to manage effective electric supply-demand balance.

The academic literature deals with a variety of models to estimate the future energy demand. A large variety of forecasting models have attempted to project electricity consumption demand by using extrapolating methods such as multiple regression models [2-4] and econometric models such as Box Jenkins methods and artificial neural networks [5]. A short-term load forecasting model has been developed using seasonal auto-regression and exponential smoothing forecasting methods employing an hourly time series load data [5]. A dynamic multivariate periodic regression model generates time-varying coefficients for trend, seasonal and regression coefficients in an hourly time series data [3]. In the researches, a significant numbers of energy demand forecasting models used the classical artificial neural networks [6, 7].

In one study, the estimated energy demand and production need in Spain was obtain using a Box-Jenkins time-series ARIMA model [9]. The multiple linear regression models is one of the popular models used in developing forecasts for electricity consumption integrated with population growth and per capita consumption rates in Turkey [10]. In a related study, energy resources policy, electric energy production and consumption rates in Turkey has been investigated and compared with that of France, Germany and Switzerland [11]. The study of electricity demand in Namibia has revealed that their electricity consumption is driven by income growth and negatively to changes in electricity price and air temperature [12]. The study of industrial, service, and residential electricity demand in Kazakhstan has shown that price elasticity of demand is low [13]. These results suggest that there is considerable room for price increases to make necessary financial adjustments and future system upgrading. In a linear regression model, the energy demand in Sri Lanka has been expressed as a linear function of price and income to examine economic growth and electricity consumption rate [14]. Another study

has investigated the impact of privatizing the power sector in Nigeria [15], revealing that privatization may reduce electricity demand if the power supply line is efficiently managed. However, the benefit of privatization may be jeopardized if the responsibility goes to inexperienced and underdeveloped private sectors. A dynamic autoregressive model has estimated short and long run electricity revenue and price in South Africa [16]. This study revealed that income significantly affects electricity demand but the electricity price rates have no effect on electricity demand. An aggregate electricity demand has been studied in South Africa [17] and the authors have concluded that growth in electricity consumption was mainly due to increased household income and improvement in the production handling factors.

Modeling household energy consumption is more complex due to high seasonal variation and constant new urbanization. Most models used in electricity demand forecast are either not capable of predicting complex demand due to high levels of demand uncertainty, or it requires large sets of data to trace the demand trend and variability to forecast. For example, a neural network model requires large amounts of data to train the model for generating sensible parameter values. Bayesian procedures are commonly adopted to forecast the electricity load [18]. Recent litrature illustrated that the Bayesian technique is applied to forecast high-dimensional parameter vectors from multivariate time series [19, 20].

3 BAYESIAN FORECASTING METHOD

This section presents a framework for an application of the Bayesian statistical technique on a dynamic linear regression model to forecast the residential electricity demand. In most cases, Bayesian inference requires numerical approximation of analytically intractable integrals [20]. Bayesian inference provides estimates of the values of unknown model parameters using some prior approximations of the parameters. These prior approximations are often expressed with the probability density functions. In order to apply Bayesian techniques, the investigators require specifying a number of subjective choices for the prior parameters. These can be determined through the iterative processes. The prior belief about the test parameters is approximated before data are collected. The choices of density models to represent the prior parameters have limited effect in determining the values of the posterior parameters. Alternately, an equally likely prior density function, known as flat prior, can be used for all parameters. Alternative priors are often used to produce different sets of posterior distributions for the test parameters. An advantage of the Bayesian approach is the ability to take into account the uncertainty of parameters in the model [21].

The procedures of Bayesian statistical application to econometric model and activity flows in steps as illustrated.

Algorithm 1: Step (1) Data preparation: Collect household electricity load data and the corresponding demographic factors related to household electricity consumption.

Step (2) Econometric model setting: Develop a classical multivariate linear regression model based on input dataset.

Step (3) Bayesian Initialization: Assign probability distribution to each parameter of regression model to form a statistical base for Bayesian application.

Step (4) Set a sampling-based iterative function: Run the statistical model using the Markov Chain Monte Carlo (MCMC) simulation method via Gibbs sampling algorithm.

Step (5) Collect samples from updated model: Obtain detail summary statistics (the mean, median, standard deviation and Bayesian confidence intervals for each parameter) from the updated samples of the posterior distribution for the values of each input variable. If the convergence of MCMC algorithm is not sufficient, go to Step 4.

Step (6) Model checking: Compare the forecast results with a classical artificial neural network model and the original multivariate linear regression model.

Bayesian statistical inference estimates unknown model parameters by combining prior knowledge and observed data to provide a probability distribution, known as the posterior distribution. The values of the posterior distribution describe the updated parameters in the model. In the Bayesian method, the values of unknown model parameters, α ($\alpha_1, \ldots, \alpha_p$) are expressed with probability density functions, $\pi(\alpha)$, which reflects the investors prior beliefs before collecting any new data. The dependence of observations $X = (x_1, \ldots, x_t)$ on model parameters α can be expressed with the probability density function as $L(X|\alpha)$, which is known as the likelihood function. The prior probability density functions are modified by the likelihood function as the new data becomes available and yield to the posterior distribution function. Using Bayes' theorem, the updating is performed as follow:

$$\pi(\alpha|X) = \frac{\pi(X|\alpha)L(X|\alpha)}{\int_{\alpha}\pi(X|\alpha)L(X|\alpha)d\alpha} \tag{1}$$

where $\pi(\alpha|X)$ is the posterior distribution and expresses the probability of the parameters after observing new data. Once the posterior distribution is available, any features of θ, such as the marginal distributions or means and variances of the individual parameter α_i, as well as the predictive distribution of future observations, require integrating over the posterior distribution. For example, the marginal posterior distribution of individual α_i is calculated as $\pi(\alpha|X) = \int_{\alpha_{-1}}\pi(\alpha|X)\,d\alpha$, where α_{-1} represents all $\alpha's$ except α_i [23]. Therefore, the forecast results in the Bayesian models is expressed by the expectation of a function of interest, $\pi(\alpha|X)$, evaluated over the posterior distribution as

$$E(\pi(\alpha|X)) = \int_{\theta_{-1}}\pi(\alpha|X)\pi(\alpha)d\alpha \tag{2}$$

where E denotes the expectation operator. In scientific literature, the limitation of Bayesian methods is the analytical solutions for the required integrations over the likelihood and the prior density functions. For Bayesian inference, limited combinations of closed form integrals for demand models and the conjugate prior probability functions are available. For most nonlinear and high-dimensional models, the inability to solve such integrals made the implementation of Bayes theorem increasingly difficult. However, with the advent of computer technology, software and the development of new methods made solving the complex integrals possible through the posterior Bayesian inference [9]. The Bayesian parameter estimation method now can be applied when the closed-form solution to a function is not available [22-23]. The method uses sampling-based approaches, particularly the Markov Chain Monte Carlo (MCMC) method and Gibbs Sampling approach to solve this problem. The flow diagram in Figure 1 shows the Bayesian process using sampling-based MCMC method.
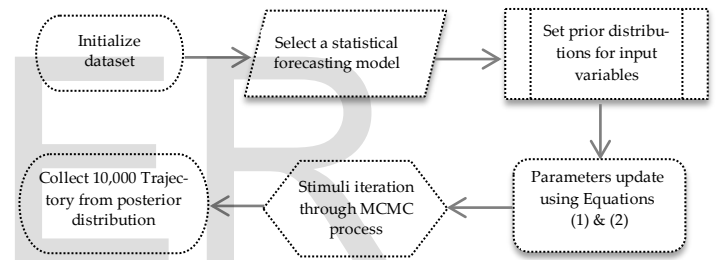

Fig 1: Bayesian Sampling-Based MCMC Method

In the Bayesian process, the method begins by selecting a probabilistic distribution for each parameter of the MLR model and observed dataset. The process requires prior probability distribution for each input variable, and then, integrates over the likelihood and the prior probability functions using the Equations (1) and (2) through the MCMC process. Input parameters are updated through simulation iteration collecting 10,000 trajectories from the posterior distribution for the values of each input variable.

4 CASE STUDY

The energy consumption pattern has a rising trend in most industrially developing countries. The innovation of modern electrical appliances, abundant availability of electronic products in the market place, increase in household income and desire for modern ways of living, together are further reasons for rapid growth of electricity demand. We use a real time series data of residential electricity demand presented by [10] to develop the forecast using the Bayesian model.

Generally, electricity demands are lumpy and vary greatly with the seasons. The demand model is integrated with five

input variables: (i) electricity prices, (2) TV price index, (3) refrigerator price index, (4) urban household size and (5) urban household income. The residential electricity demand is a function of input variables, $f(x_1, x_2, \ldots x_5)$, where $x_1, x_2, \ldots x_5$ is assigned to each input variable. The time series dataset from 1974 to 1995 is used to develop the Bayesian forecast models, and a dataset from 1996 to 2003 is used to test the efficiency of the models. First, we develop a classical multivariate linear regression model based on input dataset. Next, we assign probability distribution to each parameter of regression model to form a statistical base for Bayesian application.

Unlike regression model, the parameters of the model in Bayesian technique are considered random variable (non-static) which is flexible to accommodate any change of pattern in the data. Bayesian approac allows inserting new data (or information) into the model as it enables the estimation process to improve continuously. The Bayesian approach combines the observed data and the uncertainty of the unknown parameters to provide posterior information. The focus here is to get the posterior or marginal probability $f(\alpha|X)$ based on the likelihood $L(X|\alpha)f(\alpha)$ and the observed data. The input variables are related with the model through a link function, indicated by c_i for $i = 1, \ldots, 5$. The training model for period t, from 1973 to 1995 is expressed as follows:

$$\pi(x_1, x_2, x_3, x_4, x_5) = \mathcal{N}(\mu_t, \tau) \tag{3}$$

$\mathcal{N}(\mu_t, \tau)$ denotes as normal distribution with mean (μ_t) and precision (τ) (the receiprocal of varianece) and

$$\mu_t = \alpha_0 + \alpha_1 x_1(t) + \alpha_2 x_2(t) + \alpha_3 x_3(t) + \alpha_4 x_4(t) + \alpha_5 x_5(t).$$

Given a prior density for each parametric coefficient $\alpha_i = \mathcal{N}(\mu_0, \tau_0)$, for $i \, \epsilon \, (1, 5)$, we will obtain joint posterior distribution in Bayesian method. In the Bayesian method when prior information is not available, a common approach is to apply highly diffused probability distributions called non-informative priors. We assume $\mathcal{N}(0, 10^4)$, i.e., normal distribution with mean value 0 and inverse of variance 10^4–10^8, as diffused prior density function for each parameter. The parameter τ follows a chi squared distribution with one degree of freedom. The choice of prior distribution is followed by [6, 8]. As noted earlier, the coefficients of the input variables are estimated based on the likelihood function using the dataset from 1973 to 1995 and the prior distributions. After deriving the values for each of the coefficients, the model uses the dynamic form of the parameters to forecast electricity demand from 1996 to 2003. The forecasting model for period t, from 1996 to 2003 is expressed as follows:

$$\pi(x_1, x_2, x_3, x_4, x_5) = \mathcal{N}(\mu_t. p, \tau) \tag{4}$$

$\mathcal{N}(\mu_t. p, \tau)$ is normal distribution with mean ($\mu_t.p$), where
$\mu_t. p = \alpha_0 + \alpha_1. p x_1(t) + \alpha_2. p x_2(t) + \cdots + \alpha_5. p x_5(t).$
The estimates obtained with the Bayesian techinique derive from the difference between the mean of the sampled values

(estimate of the posterior mean for each parameter) and the true posterior mean [20].

5 FORECASTING RESULTS OF BAYESIAN MODEL

Modeling household energy consumption is more complex than the regional energy consumption model due to demographic factors, modern household electronic appliances and growing income. Bayesian approaches are the preferred methods capable of modeling nonlinear and non-stationary time series demand. In this forecasting model, given the time series data, a Monte Carlo Markov Chain (MCMC) is constructed using the Gibbs sampler algorithm for obtaining posterior inference of input parameters through simulation. The summary statistics such as mean, median, standard deviation and Bayesian confidence intervals for each parameter are estimated using the MCMC samples from the posterior distributions in the Bayesian procedure. The sample mean and standard deviation (SD) of each parameter under Bayesian process is in Table 1.

TABLE 1
BAYESIAN MODEL PARAMETER ESTIMATES

Parameters	α_0	α_1	α_2	α_3	α_4	α_5
Mean	4.33	143.9	-0.619	78.03	230	108.5
SD	31.72	128	122.6	106.6	151	109

Using the parameter values presented in Table 1 and corresponding density functions, the forecast obtained by mean values of posterior function and 95% confidence level is shown in Table 2. The proposed forecast result compares with multivariate regression model and neural network model.

TABLE 2
FORECAST RESULTS & COMPARISON

Year	Actual	Bayesian statistica technique			Regresion model	Neural network
		Mean	2.5%	97.5%		
1996	23993	27770	21660	34320	25276.8	25083.2
1997	26523	26920	21060	32930	32224.6	28146.1
1998	28686	27750	21740	33610	32441.5	32170.8
1999	29754	30780	24020	37620	33924.8	29426.4
2000	31266	31730	25070	38140	34703.4	31651.8
2001	32891	32940	26370	39730	39912.8	32764.4
2002	34946	35110	28250	41740	39824.4	35563.7
2003	37967	37680	29680	45280	35353.9	36963.8

A public domain software package, WinBUGS, [22] is used to generate feasible posterior samples via MCMC simulations in minimum computational time. The convergence of the MCMC algorithm is checked by the ANOVA-based method integrated in WinBUGS [23]. Trace show sampled value at each iteration

in Figure 2. In 'chain', there is no particular pattern nor exists any auto-correlation.
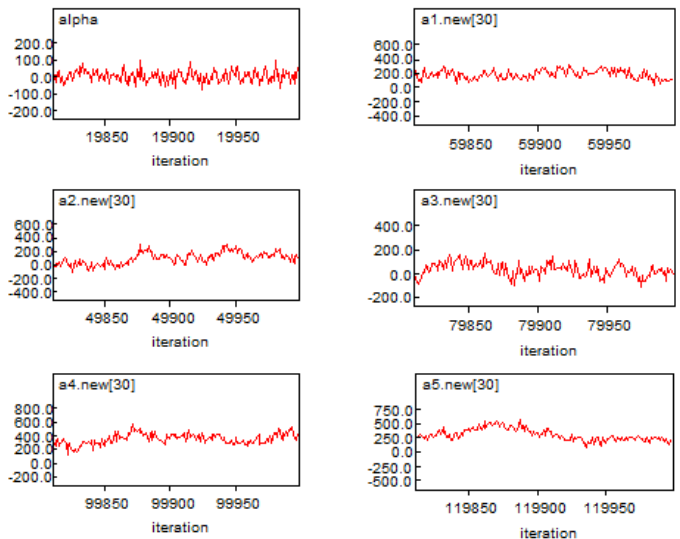

Fig 2: Trace of sampled value at each parameter

The density plots of the prior distribution and the posterior estimates are presented in Figure 3
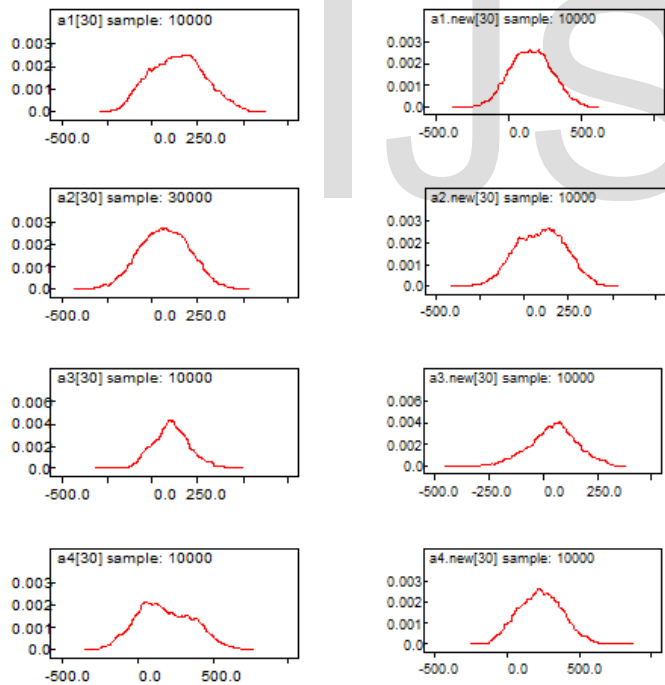

Fig 3: Density estimates plots of the parameter

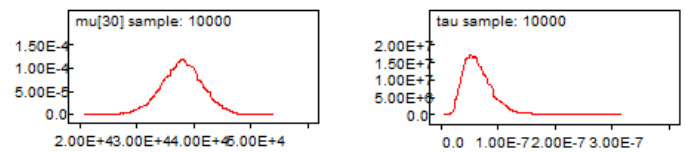The plots of the density estimates of the mu (μ) and tau (τ) are illustrated in Figure 4.

Fig 4. Density estimates plots of mu and tau

6 MODEL COMPARISON

Sufficient and adequate production of electricity is a necessary condition for achieving sustainable economic development [24]. The proposed Bayesian model applied to MLR is tested with regression and neural network model forecasts. In the forecast, X_f is the fit of model. X_{obs} is the real observatoin. Therefore, error, $E_t = (X_{obs} - X_f)$. The mean absolute error deviation $(MAD) = 1/n \sum_{t=1}^{n} |E_t|$. The mean absolute error deviation is the average of the absoutule deviation over all periods. The percentage error (P_{error}) is absolute error over the real observation. The forecast index including forecast percentage error (P_{error}) and MAD is illustrated Table 3.

TABLE 3
FORECAST ERROR COMPARISON

Year	P_{error}			MAD		
	Bayesian technique	Regresion model	Neural nework	Bayesian technique	Regresion Model	Neural nework
1996	0.157	0.054	0.045	3777.00	1283.8	1090.2
1997	0.015	0.215	0.061	2087.00	3492.7	1356.6
1998	0.033	0.131	0.121	1703.33	3580.3	2066.0
1999	0.034	0.140	0.011	1534.00	3727.9	1631.4
2000	0.015	0.110	0.012	1320.00	3669.8	1382.3
2001	0.001	0.213	0.004	1108.17	4228.5	1173.0
2002	0.005	0.140	0.018	1362.60	6049.9	1531.2
2003	0.008	0.069	0.026	1183.33	5477.1	1443.2

In the comparison, the result for classical neural network model is borrowed from [10]. The forecast developed by Bayesian technique and neural network models are compatible. However, forecast by the classical multivariate method is not comparable with the above models. The result derived by the Bayesian model is marginally better than those from the neural network model. Following are the error index: the mean square error $(MSE) = 1/n \sum_{t=1}^{n} E_t^2$; the mean absolute percentage error $(MAPE) = 1/n \sum_{t=1}^{n} |E_t/D_t| * 100$. the mean absolute percentage error is the average absolute error as a percentage of demand. Table 4 illustrates the comparison of forecast performance between the Bayesian application to regression model with that of neural network and classical multivariate regression models.

TABLE 4

PREDICTION PERFORMANCE COMPARISON

Error Index	Bayesian technique	Regresion model	Neural nework
MSE	0.0208	0.1968	0.022
MAPE	0.0335	0.1339	0.0351
MAE	0.0089	0.0411	0.0108

Numerical values of the forecasting error index such as the mean square error, mean absolute percentage error, and mean absolute error show that the Bayesian technique applied to regression model exhibits less error than of the neural network model and the multivariate regression model. Therefore, Bayesian technique appears to be the most appropriate to recognize underlying demand pattern and suitable for enterprise forecast practices. It is understood that judgment-based forecasting procedures are not adequate to draw a fair forecast conclusion for a dataset with multiple variables. In contrast, simply replacing the management judgment about future events with advanced data extrapolation methods, such as neural network or regression, with no judgmental input is also unrealistic. Although the Bayesian model requires subjective choices, one can find more subjective choices in developing a neural network model than a Bayesian model. The Bayesian model requires less data compared to many econometric and neural network models.

A. Tracking Signals

The tracking signal indicates a bias in the forecast. Forecasters need to monitor a forecast to determine when it needs to change or update the model. Tracking signal is measured by $1/n \sum_{t=1}^{n} E_t / |E_t|$. It measures whether the forecast reflects the actual demand with respect to the level and trend in the demand profile. Similar to statistical process control, the tracking signals measure when the forecasting process is going out of control and it needs intervention for adjustment. Tracking signals also indicate if there is a persistent tendency of forecasts to be higher or lower than the actual demand. If the forecast is consistently lower than the actual demand, then there is under forecasting and the tracking signals will display only the positive trend. In the best forecast, the tracking signals move along the zero axes, going upward and downward. The tracking signals of all forecast model must pass a threshold value, i.e., above 3.75 or below -3.75. Otherwise, the forecast indicates an extreme bias. The tracking signal of Bayesian model passes with ±0.85, while the tracking signal of neural network model follows very similar, but opposite trend. Tracking signal of regression model runs below zero. Tracking signal comparison of the forecasting models is shown in Table 5.

TABLE 5

FORECAST TRACKING SIGNALS

Models	1996	1997	1898	1999	2000	2001	2002	2003
Bayesian Technique	↗	↗	↑	↘	↗	↑	↘	↑
Neural Network	↘	↘	↑	↘	↘	↑	↘	↗
Regression Model	↘	↘	↗	↘	↗	↘	↘	↗

where ↑: positive upward; ↗: negative upward; ↘: negative downward.

Observing the forecast performance, it is evident that all tracking signals are within the limit. However, forecast tracks of Bayesian application to regression model show unbiased trend relative to that of the neural network model and the regression model. Clearly, the regression model provides consistenly negative signals, indicating enormously biased forecast. The application of the Bayesian technique on the regression model in a time series improved the quality of the residential electricity demand forecast. The improvement happens due to flexibility of the Bayesian technique to capture the dynamic changes of the trend as the new data and experts' observations become available.

7 Conclusion

The need for accurate projection of electricity demand is crucial; hence, it is of vital interest to choose the best forecasting model. Forecast of rapidly growing residential electricity demand is a complex task, particularly in industrially developing countries. The government sector, electric power industry and energy divisions require advanced planning to manage energy supplies for wider usages. The proposed model uses the Bayesian technique to illustrate a residential electricity demand forecasting problem. The study explored the potential advantage of the Bayesian technique applied to econometric forecasting models to improve the forecast results. For a given set of electricity demand data, the model is compared with an artificial intelligence method and a classical statistical forecasting model. For an 8-year forecast period, the results clearly indicate that values forecasted by the Bayesian model are fairly close to the real observations, compared to those of neural network and regression models. Tracking signal evaluation also specified the unbiased depiction of the Bayesian model. The dataset used here shows annual changes of demand pattern and nonlinear trends; however, the Bayesian method has improved the predicting capability of data patterns and the ranges of variability. The forecast result significantly reduces the variability and the errors. The technique is suitable for enterprise forecast practices to predict the future.

The Bayesian technique appears to be flexible, comprehensive and accessible to incorporate the experts' judgment about future trend of the demand. Since the Bayesian model can captures rapid changes in the demand, it performs better than other econometric and artificial intelligent forecast models. Energy demand often comprising multiple input variables, such as demographic factors, industrialization factors and consumer income pattern. The illustrated Bayesian technique can be used as an effective tool to improve any econometric forecasting model to project the future trend of energy usages. There is only a handful of forecasting methods capable of predicting rapidly growing and large fluctuating demand. In most cases, practitioners do not follow the most recently developed software and technological innovations. Software packages for Bayesian statistical application are generally open sourced. It is commonly perceived that these software are user friendly, easier to implement in developing complex models. The technique is capable of combining expert judgment with the latest data information in the model process to find the best forecast solutions. Although there are many favorable features available in the Bayesian methods, these models are still much less frequently used for forecasting the future of energy demand.

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